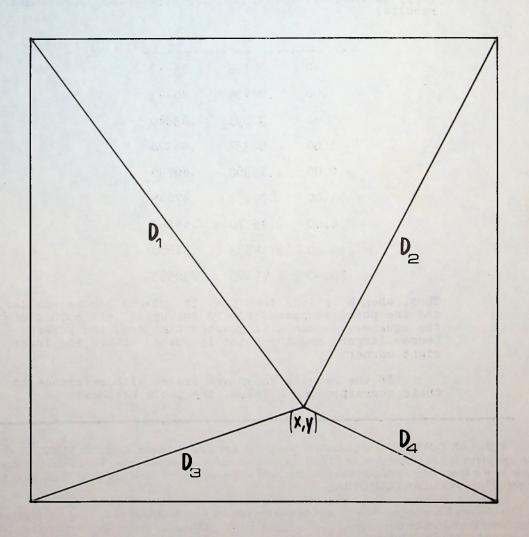
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November 1977 Volume 5 Number 11

Popular Computing



A Problem of Scale

A Problem of Scale

In the Figure on the cover, point (x,y) is to be located in the square of side K so that

$$D_1 + D_2^2 + D_3^3 + D_4^4 = F$$

is a minimum. For any given value of K, the point (x,y) is uniquely located. We have these approximate results:

K	ж	У
.40	.08193	.34473
.60	.24434	.40143
.80	.38891	.43840
1.00	.52337	.46320
2.00	1.15360	.49790
3.00	1.79253	.47645
4.00	2.45770	.44553
5.00	3.14553	.41680
10.00	6.79625	.32583

Thus, when K is less than one, the powers become smaller, and the point is pushed toward the upper left corner of the square. When K is greater than one, the powers become larger, and the point is pushed toward the lower right corner.

If the results shown are scaled with reference to their corresponding K value, the table becomes:

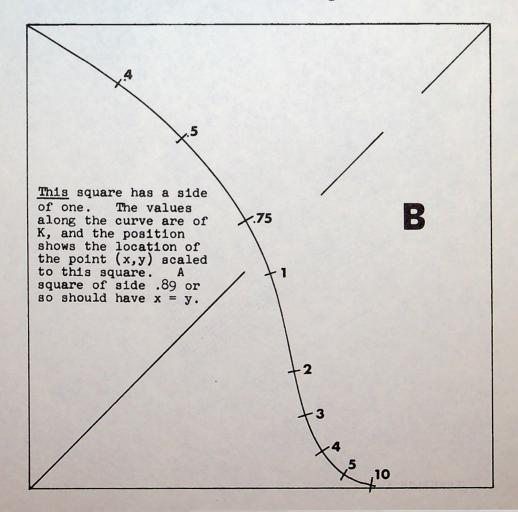
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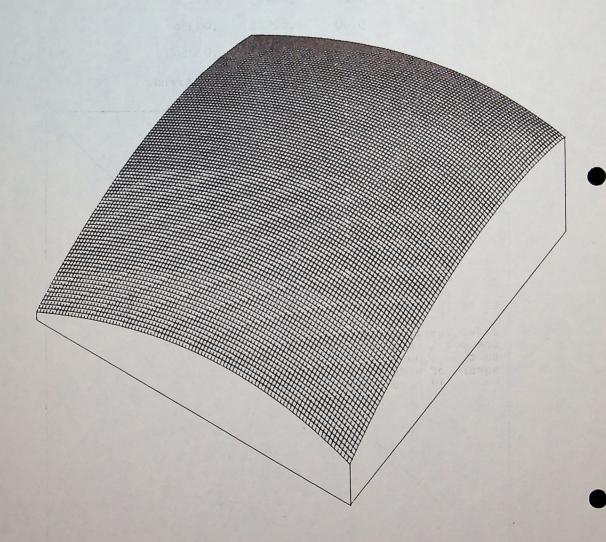
х	У
.20483	.86183
.40723	.66905
.48614	.54800
.52337	.46320
.57680	.24895
.59751	.15882
.61443	.11138
.62911	.08336
.67963	.03258
	.20483 .40723 .48614 .52337 .57680 .59751 .61443

Figure B below shows the resulting trend.



There are three problems here:

- 1. For a given value of K, is there an analytic way to determine the point (x,y)?
- 2. What is a practical search method to find the point (x,y) for a given value of K? That is, what process corresponds here to bracketing, or interval-halving?
- 3. It is clear from Figure B that, for some value of K, x = y. How can that value of K be found?



The function whose minimum we are seeking is quite well behaved. For any particular value of K, say K = .5, we can calculate the value of the function for 10,000 points (that is, 100 equally-spaced values on x and y) as shown in the accompanying plot, made by Associate Editor David Babcock. (The function has been inverted, so that we are looking at the under side, where the true minimum now sticks up as the highest point.) Thus, any systematic search procedure should not run into problems of local minima, saddle points, and such.

Mr. Babcock used a bracketing process to locate x and y to 8 significant digits for a given value of K. From the following table:

К	sa riven	et as very sente	function
.81	.39582310	.43988511	1.19090567
.82 .83	.40271230	.44134320	1.21965054
.84	.41642190	.44417459	1.27823601
.85 .86	.42324380 .43004460	.44554880 .44689610	1.30808116 1.33829760
.87 .88	.43682490 .44358538	.44821710 .44951220	1.36888758
.89	.45032661	.45078190	1.43119716
.90	.45704920	.45202660 .45324690	1.46292124
.92	.47044080	.45444310	1.52751910
.93	.47711080	.45561570	1.56039733

it appears that around K = .89, x and y will be equal. That is our problem 211: to find that value of K to greater accuracy.

This PROBLEM OF SCALE is one of the most challenging problems we have presented. Various schemes for locating (x,y) for a fixed value of K by mathematical approaches have not proved feasible, and no scheme (other than cutand-try) suggests itself for finding the K for which x = y. We solicit further investigation by readers.

Problem 187 (issue 54) called for polynomial equations with integral coefficients having a root close to pi. Herman P. Robinson calls attention to these:

 $19x^2 - 39x - 65 = 0$

3.1415882...

99532x = 312689

3.14159285...

 $x^4 = 81 + 361/22$

3.1415926526...



In issue 53, page 11, a list of corrections for the logarithm table of issue 51, page 15, was given.

Unfortunately, that list contained an error, too.

The correct value for the logarithm of 59 (the 41st through 50th decimals) is:

46789 33045



Problem 189 (issue 54) called for the formation of a transcendental number, GG, in which the Kth decimal place of GG is the Kth decimal in the square root of K.

Herman P. Robinson calculates GG to be:

.01206 93201 57946 04621 86380 04248 59904 00321 24398 90104 34357 08437 75706 26057 00364 30259 03888 40185 47396 07020...



Hamming's Law of Statistics

90% OF THE TIME THE NEXT *INDEPENDENT* MEASUREMENT OF SOMETHING WILL FALL OUTSIDE THE REPORTED 90% CONFIDENCE LIMITS

So you think you understand how random processes will behave?

Exploring Random Behavior -- 3

We have been exploring the contention that people acquire a feeling of confidence concerning the nature and behavior of random numbers but do not really understand how they behave. This month we present a little self-test.

As usual, we assume the availability of a random number generator whose output is a long stream of numbers, X_i , uniformly distributed in the range from zero to one, but not including those values; that is,

$$0 < X_{i} < 1.$$

Any large collection of such numbers should have a mean of .5 and a standard deviation of .288675.

What sort of distribution is found by taking various combinations of such numbers? In the ten combinations at the right, each X represents a separate random fraction. The game here is to guess the mean and standard deviation of a set of numbers formed as indicated.

For example, in case 1 we have $\frac{X+X}{2}$.

This means that two random numbers are drawn, and the average of the two is taken. Call one such average value R. For a set of many values of R, what will be the mean and standard deviation?

Estimates are given on the next page, but the reader is urged to make his own estimate first.

$$\frac{x+x}{2}$$

$$\frac{x+x+x}{3}$$

$$\frac{x+x+x+x}{4}$$

$$4 \frac{x + x}{x + x}$$

$$5 \sqrt{x^2 + x^2}$$

$$6 (\sqrt{x} + \sqrt{x})^2$$

$$7 \qquad \sin^2 x + \cos^2 x$$

$$8 \sqrt[-x]{x^x}$$

$$Q \qquad ((xx + x)x + x)x$$

$$\frac{x}{x}$$



From limited empirical runs, we have the following results:

Type	Mean	Standard Deviation
1	.4862	.2050
2	.5100	.1698
3	.4938	.1443
4	1.6414	2.3890
5	.7832	.2845
6	1.8458	.8409
7	1.0019	.3176
8	.5053	.3336
9	.4281	.3315

Type 10 (X/X) is the hard one. Even if a given X cannot be zero, a combination like

$$\frac{.789543}{.001234} = 639.8$$

must be expected occasionally, whereas the "normal" ratio will average 3.5 or so. Successive runs in which any ratio that exceeded 300 was excluded from the calculation showed these results:

(each of these runs was of 100 trials.)

D. 1.1 201 (1 5h)
Problem 191 (issue 54) was titled "A Coding
Exercise," and was intended to be just that.
The function, X, is to advance by D, with D
being successive integers, each repeated the
number of times (1, 1, 2, 3, 5, 8, 13,) given
by the Fibonacci sequence. Problem 191 asked
for the 1000th term of the X sequence, and a
formula for the Nth term.
Tormula for the Nth term.

A flowchart is shown for a direct attack on the problem.

David E. Ferguson has produced a formula for the Nth term. For \mathbf{X}_{N} , determine n from the Fibonacci sequence such that

$$u_n \leq N < u_{n+1}$$

Then $X_N = (n - 1)N - u_{n+2} + 3$.

Thus, for X_{1000} , we first determine the position of the number 1000 in the Fibonacci sequence (shown at the right) as between terms 16 and 17, giving n=16. Then

$$x_{1000} = (15)(1000) - 2584 + 3 = 12419.$$

Direct calculation, using the logic of the accompanying flowchart, is in agreement.

Ferguson's formula lets us calculate any term of the required sequence with minimum effort. For the 100,000th term, for example, it is easy to find that the number 100,000 lies between the 30th and 31st term of the Fibonacci sequence, so that n=30. Then

$$x_{100000} = (29)(100000) - 2178309 + 3$$

= 721694

1	1	1
2	2	
3	4	2
4	7	3
		3
5	10	4
6	14	4
7	18	4
8	22	
9	27	5
1		

X

N

D

1 1 2 1

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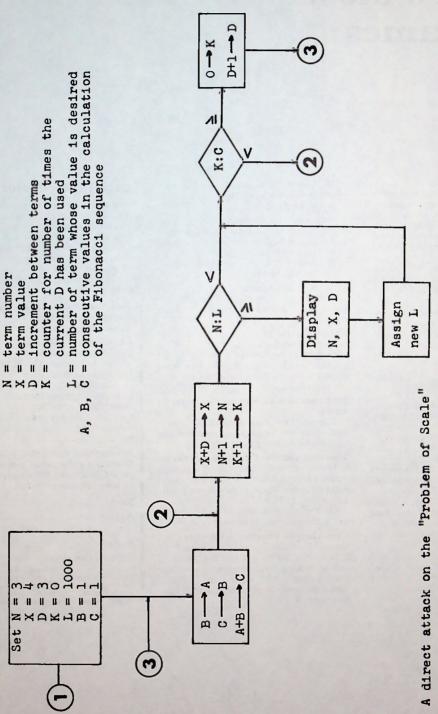
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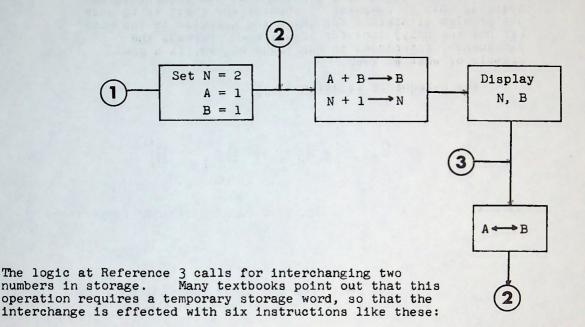
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sequence. At Reference 3, the next term of the Fibonacci sequence is calculated.

A simple scheme for generating the Fibonacci sequence is shown here.



LDA A
STØ TEMP
LDA B
STØ A
LDA TEMP
STØ B

Professor John Motil, California State University, Northridge, shows that the extra word of storage is not necessary. The logic of an interchange can be:

$$B - A \longrightarrow B$$

$$A + B \longrightarrow A$$

$$A - B \longrightarrow B$$

and this can be done in seven instructions:

LDA	В
SUB	A
STØ	B
ADD	A
STØ	A
SUB	В
STØ	В

--a classic example of swapping data storage for instruction storage.



But going back to Problem 191, Mr. Ferguson has again demolished analytically a problem that was thought to be solvable only by computer. One of our goals is to seek out problem situations for which the computer is the best (if not the only) tool for solution. Perhaps the Z-Sequence, introduced in our issue No. 42, is a good example of what we seek.

The Z-Sequence is defined by:

$$a_n = |2a_{n-2} + a_{n-1} - n|$$

with $a_1 = a_2 = 1$. This sequence has interesting properties:

- Successive terms seem unpredictable; that is, the values jump around, but not too wildly.
- 2. In the long run, 1ts terms tend toward n/3, where n is the term number. Thus, the sum of the first 8002 terms is 11245744, whereas the sum of n/3 for n = 1, 2, 3,...,8002 is 11213501, a difference of only 32233.
- 5. The sequence is readily generated in any language on any machine, using the simplest of operations, and dealing only with positive integers. The accompanying table gives restart values at various points in the sequence.
- 4. No term after the first is ever as large as n. The circled numbers in the original table (see PC42-13) show the appearances of new larger values. Such new larger values are seen to occur 68 times in the first 5081 terms.
- 5. Zero appears at terms 3, 7, 11, 28, 31, 140, 239, 600, and 6476, but does not seem to appear after that.

 Up to n = 5000, only 67 terms have a value less than 5.
- 6. Successive equal terms appear at n = 90 and n = 98, but this phenomenon does not seem to occur again.



- 7. If item 2 above holds indefinitely, then the sum of the series should be approximately the sum of n/3. The sum of each term divided by n should be approximately n/3 itself. The sum of each term divided by the square of the term number should be approximately 1/3 the sum of the harmonic series (see PC9-14). The sum of each term divided by the cube of the term number, however, should approximate 1/3 the sum of (1/n²), which should converge. It appears to converge to 1.2346...
 - 8. Almost every integer appears as a term in the sequence sooner or later, except for a few numbers, such as 245, 449, 569, 575, and 903.

A great deal of information has now accumulated about the Z-sequence. Table B shows the frequency, for every 1000 terms, of terms that are less than 1000. Although the trend is for terms to approximate n/3, where n is the term number, it is seen that small values persist with remarkable stability.

Zero terms occur well beyond the point (term 6476) first indicated. The following terms are zero:

3, 7, 11, 28, 31, 140, 239, 600, 6476, 33172, 64375, 65287, 79051, 97864, 105099.

and successive equal terms occur again at 5518 and 40722.

Table C shows the appearance of each integer from 000 to 499 for the first time in the sequence. Only 22 numbers are yet unaccounted for, from 000 to 999.

Table D shows restart values for the sequence, up to term 100,000. For each entry in the table, the last number is the term number (e.g., 10,000) and the other three are the term values at that term and the two that precede it. Thus, at 10,000, the table shows

term 9998: 4823 term 9999: 3634 term 10000: 3280

2000 8 3000 6 4000 4 5000 3 6000 3 7000 2 8000 2 9000 2 10000 1 12000 1 15000 1 15000 1 16000 1 17000 1 18000 1 19000 1 20000 1 21000 9 22000 23000 24000	000 26000 83 27000 42 28000 80 29000 96 30000 67 32000 62 33000 12 34000 13 35000 95 36000 78 37000 68 38000 46 39000 36 40000 29 41000 15 42000 24 43000 14 44000 08 45000 3 46000 95 47000 94 48000 92 49000 75 50000	61 57 63 56 49 62 46 53 73 541 41 45 340 38	55000 56000 57000 58000 59000 60000 61000 62000 64000 65000 66000 67000 67000 67000 71000 72000 74000 75000		76000 77000 78000 79000 80000 81000 82000 83000 84000 85000 86000 87000 98000 91000 91000 92000 93000 94000 95000 96000 97000 98000 99000	25 27 22 18 17 18 29 22 24 26 24 30 20 16 15 23 21 21 17 30 21 16 14 15 24
25000			75000	25]		
	101. 1 016	BM TO	u . ollo,	1 00	, nevert	411 1871:

734.2 6.0 Fable D.5 6.0 6476, 105099

167	99	1545	1953	2253	767	2517	7807
194	462	240	1404	340	4654	3260	22
472	60	330	1310	154	188	1294	7636
1000	2000	3000	4000	5000	6000	7000	8000
1291 7422 1004 9000	4823 3634 3280 10000	1655: 6818 0 872 11000	5753 2080 1586 12000		8309 4884 7502 14000	2778 152 15000	16000
7847	2659	3833	12913	14039	6417	2669	15729
7050	6330	1024	5768	1806	11276	3968	3752
5744	6352	10310	11594	8884	2110	13694	11210
17000	18000	19000	20000	21000	22000	23000	24000
13021 7712 8754 25000	16787 7294 14868 26000	17854 7820 27000	7277 2348 11098 28000	9099 6174 4628 29000	1991 2686 23332 30000	3871 16290 6968 31000	5747 4698 15808 32000
15177	4655	23449	7149	3915	26897	6115	12507
7532	25758	3480	26576	32954	7328	7274	21946
4886	1068	15378	4874	3784	23122	19496	6960
33000	34000	35000	36000	37000	38000	3 9000	40000

Table D--Continued

1.77		. 30					
11033	12245	16565	16315	2395	4471	4467	24751
26000	15080	18364	26798	5934	33806	23014	11270
7066	2430	8494	15 42 8	34276	3252	15052	12772
41000	42000	43000	44000	45000	46000	47000	48000
6315	513	14769	341	31331	32831	23075	16751
25772	28596	3328	4264	8570	12954	21882	2638
10648	20378	18134	47054	18232	24616	13032	19860
49000	50000	51000	52000	53000	54000	55000	56000
4541	26239	9295	2077	20275	21557	36101	23439
12576	8814	5146	30104	34978	596	19060	21146
35342	3292	35264	25742	14528	18290	28262	4024
57000	58000	59000	60000	61000	62000	63000	64000
14115	18927	11663	30495	43671	36919	16137	47779
20890	35038	25794	30014	10822	23638	16648	9126
15880	6892	17880	23004	29164	27476	22078	32684
65000	66000	67000	68000	69000	70000	71000	72000
5805	31545	45139	36979	13545	4535	64929	2473
29432	2468	28170	2702	35732	46182	7888	43484
31958	8442	43448	660	14178	22748	58746	31570
73000	74000	75000	76000	77000	78000	79000	80000
27615	17787	16037	17871	35333	67975	37701	43
45682	16682	53140	198	10756	10922	21168	41726
19912	29744	2214	48060	3578	60872	9570	46188
81000	82000	83000	84000	85000	86000	87000	88000
4265	24787	51869	39035	10421	47615	59959	11837
57984	33974	8684	15606	2028	37306	17846	72904
22486	6452	21422	1676	70130	38536	42764	578
89000	90000	91000	92000	93000	94000	95000	96000
82425	21127	33709	12369	52201	172359	26473	170955
1136	36478	47796	27604	47992	14606	89536	86366
68986	19268	16214	47658	2394	159234	107518	1282 7 6
97000	98000	99000	100000	150000	200000	250000	300000

Table C

	0	1	2	3	4	5	6	7	8	9
00 01 02 03 04 05 06 07 08	3 20 44 139 168 112 256 163 143 408	6 13 73 165 61 294 142 529 117 293	4 55 75 72 147 119 295 127 167 115	5 77 69 121 146 129 190 145 270 274	35 27 64 535 59 347 79 107 223 188	10 70 34 97 93 138 101 162 2979 218	8 56 32 83 123 124 271 200 203 228	26 53 585 253 286 81 109 174 678 730	15 36 51 67 815 144 136 460 316 551	38 282 30 469 1398 194 445 478 157
10 11 12 13 14 15 16 17 18	423 287 320 208 592 416 484 319 367 960	186 3188 1521 690 489 1074 266 341 798 562	1028 1348 219 296 184 407 455 3975 656 243	177 201 3609 1078 314 182 962 326 402 245	336 160 307 359 980 291 1607 639 412 999	229 213 193 257 313 226 206 285 733 301	132 252 667 356 268 216 255 892 351 532	290 446 153 577 393 537 277 1573 505 741	155 315 2604 236 247 732 487 399 875 324	433 2252 249 321 2538 426 1537 1093 986 1441
20 21 22 23 24 25 26 27 28 29	392 264 1223 723 260 695 2208 424 2808 599	834 241 654 310 1893 346 693 454 2590 1018	7512 968 467 1215 619 1979 560 584 515 1260	697 1285 637 734 538 2193 1022 750 1650 497	1135 283 380 672 4680 1891 419 904 4252 475	942 410 262 4082 397 2857 389 885 882	364 1552 1212 1340 895 1128 1155 452 1932 680	477 794 1597 533 773 362 518 1221 441 422	243 339 1071 1160 332 1036 5088 432 344 883	3229 334 337 354 1566 1030 782 482 430 378
30 31 32 33 34 35 36 37 38 39	803 439 376 1812 707 1791 1908 4135 731 3267	761 742 3997 1561 581 3218 649 2382 1850 2277	771 536 1396 1159 952 751 508 4444 1419	470 1741 1433 1226 374 858 450 1026 1126 545	748 2279 1496 675 2135 1279 1316 992 495 576	698 549 5450 737 1166 2178 754 797 1921 1393	387 1556 996 1600 792 1456 1251 628 1356 543	510 657 4890 554 578 513 1218 670 1137 1814	480 916 2239 596 372 851 523 547 907 1332	4101 793 1290 530 525 370 437 1458 846 1174
40 41 42 43 44 45 46 47 49	696 1559 647 747 1696 1184 1575 5588 3591 1380	3510 1261 721 1258 521 785 2821 1005	2496 1068 804 2264 4643 1167 1335 1000 1324 2611	1382 617 694 922 953 1053 1670 1517 1037	3247 687 612 816 2927 1720 1104 823 3139 2603	574 749 1125 2201 1581 2406 1678 1797 3785 890	591 552 1528 2180 1432 2484 2008 1015 8891 2867	2501 493 1162 1142 685 1354 1861 3381 589 2094	1288 8747 640 1852 2299 1016 716 764 668 3347	1849 865 2085 826 2674 541 1278 969 610

That South American Roulette System

If you can obtain different odds on the same event, then you can guarantee a win for yourself by adjusting two bets. For example, suppose that you can obtain odds of 2:1 that A will win over B from one source, and odds of 3:1 from another source. You could bet as follows:

	if A wins	if B wins
Bet \$6 that B wins at 2:1	-6	+12
Bet \$6 that A wins at 3:1	+18	-6
net:	+12	+6

The situation was stated in extreme terms, but the principle is the same no matter how close the odds are, as long as they are different.

In general, given an event for which the odds on A winning are P and Q (and assuming that the P odds are better than the Q odds), then one could bet

$$PX - Y = \frac{Y}{Q} - X$$

where X and Y are the amounts to be bet. Then:

$$Y = \frac{1/P + 1}{1/Q + 1} X$$

giving the ratio of the bets. For example, if odds are given of 8:7 and 9:8 on A winning,

$$Y = \frac{8/7 + 1}{8/9 + 1} X$$

$$Y = (135/119) X$$

and one could bet 119 units on A with P odds, and 135 units on B with Q odds. Whether A or B wins, you collect one unit.

The so-called South American roulette system is based on these considerations. See the layout of the American roulette wheel. Odds of 1:1 are offered on either red or black. Odds of 2:1 are offered on each of the three columns of numbers. It is seen that we have:

column 1: 6 red, 6 black

column 2: 4 red, 8 black

column 3: 8 red, 4 black.

Consider column 1, with red and black evenly distributed. The chance of getting black in column 1 is thus 1:1, but the chance of landing in that column is 1 out of 3 (neglecting the chance of landing on zero or double zero). It appears then that we are presented with differing odds on the same event. This also appears to be true for the second and third columns. It would seem that some combination of betting simultaneously on red/black and on one of the columns would guarantee a win. (It won't, but you can have fun trying.)



When you have devised your winning system, you can play it by simulating a roulette wheel with a computer program and playing the system at high speed and low cost. A real-life wheel runs about 500 plays during an 8-hour shift and requires money to play. A computer-simulated wheel should run about 500 plays per minute, and you can set your own stakes and your own house limit.